

## NOTATION

$q_{cr}$ , critical thermal flux density;  $p(x)$ , function which considers distribution of specific thermal flux  $q$  over height of vapor generator channel;  $x$ , coordinate;  $\rho_c''$ , vapor density above vapor generator channels;  $\rho'$ ,  $\rho''$ , mean density of liquid and vapor over channel heights;  $\rho''$ , local vapor density;  $r$ , specific heat of vapor formation;  $t_s$ , saturation temperature at input to vapor generator channel;  $\bar{h}_0$ , mean height of liquid column in descent tube over water input to vapor generator channel.

## LITERATURE CITED

1. A. P. Simonenko, "Study of heat liberation in cooling agent boiling in vertical tubes," Tr. Nikolaev. Korablestroï. Inst., No. 187, 19-26 (1982).
2. N. V. Tarasova and A. I. Leont'ev, "Hydraulic resistance in flow of a vapor-water mixture in a heated vertical tube," Teplofiz. Vys. Temp., 3, No. 1, 115-119 (1965).
3. O. O. Mil'man, "Limiting thermal loads in boiling in heat exchangers with vertical tubes," All-Union Conference on Thermophysics and Hydrodynamics of Boiling and Condensation Processes. Summaries of Reports [in Russian], Vol. 1, Ryzhsk. Politekh. Inst., Ria (1982), pp. 186-188.
4. S. S. Kutateladze and M. A. Styrikovich, Hydrodynamics of Gas-Liquid Systems [in Russian], Énergiya, Moscow (1976).
5. M. K. Bezrodnyi et al., "Study of limiting heat transfer of inclined two-phase thermosiphons with separation of ascending and descending flows of intermediate heat transfer agent," Promyshl. Energet., No. 5, 44-48 (1982).
6. V. E. Doroshchuk, Heat Exchange Crises in Water Boiling in Tubes [in Russian], Énergoatomizdat, Moscow (1983).

## CALCULATION OF DIFFUSION SEPARATION PROCESSES IN GAS MIXTURES

E. P. Potanin

UDC 533.735

The selective action of various types of force fields on isotopic gas mixtures is considered using the multicomponent hydrodynamic approximation. It is shown that it is possible to indirectly estimate the intensity of mutual diffusion and the separation effect in all cases of practical importance.

Calculation of the degree of enrichment of an isotopic gas mixture achievable in an individual separation device involves analysis of mutual diffusion of the components under the action of various types of force field [1-8]. Separation may be produced by the selective action on the mixture of not only purely external forces, produced by, for example, "gravity-like" (centrifugal [1, 4], gravitational) or electromagnetic fields [2], but also forces of an internal nature, among which, in particular, are viscous forces [3], as well as diffusion friction forces [4-6]. Multiple separation processes are found in plasma devices (the plasma centrifuge [4, 9, 10], traveling magnetic wave system [5, 11], dc discharge [12, 13]), in which several separation mechanisms may operate simultaneously. Among such mechanisms, in particular, are thermodiffusion and the centrifugal effect, mass diffusion and mechanisms related to the differing degree of ionization of the components. A recent analysis of enrichment processes in plasma systems permits formulation of a simple general method for calculating diffusion separation phenomena in the presence of a pressure gradient within the gas.

To determine the mutual diffusion rate of the components of a binary gas mixture we will use the multicomponent hydrodynamic approximation [4]. The equations of equilibrium of the volume forces acting on the mixture components can be written for the general case in the form

---

Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 48, No. 4, pp. 628-632, April, 1985. Original article submitted March 21, 1984.

$$-\frac{\partial p_1}{\partial x} + n_1 f_1 = n_1 n_2 a_{12} (v_1 - v_2), \quad (1)$$

$$-\frac{\partial p_2}{\partial x} + n_2 f_2 = n_1 n_2 a_{12} (v_2 - v_1), \quad (2)$$

where  $f_1$  and  $f_2$  are "external" forces, acting on the molecules of each type. By external we understand forces of any origin which are responsible for the development of a pressure gradient in the mixture being separated. Such forces may be: a) purely external forces, produced by the direct action on the molecules of gravitational or electromagnetic type force fields; b) inertial forces, related, for example, to centrifugal or other acceleration; c) internal forces among which are viscous forces as well as diffusion friction. We note that the latter are not forces in the general meaning of the term. They are average values defined as the fraction produced by dividing the force acting on a unit volume of the given component by the numerical density. At the same time, as will be shown below, introduction of such quantities is very convenient in calculating separation effects in mass-diffusion systems [1, 4-6] and in the case of mixture flow in channels [3, 7]. We stress especially that mutual friction between components of the separating mixture, described by terms on the right side of Eqs. (1) and (2) and referring to discharge of diffusion friction forces are not an "external force, since they do not affect the pressure gradient of the separating mixture.

In analyzing separation processes in gas mixtures the physically justifiable formulation of the problem usually assumes the presence of impermeable walls bounding the diffusion zone, as a result of which we require a condition of equilibrium for the mixture as a whole:

$$\frac{\partial p}{\partial x} = n_1 f_1 + n_2 f_2. \quad (3)$$

Using Eqs. (1)-(3), we find for the difference in mean velocities

$$v_1 - v_2 = -\frac{D_{12}}{\gamma(1-\gamma)} \left\{ \frac{\partial \gamma}{\partial x} - \alpha_p \frac{1}{p} \frac{\partial p}{\partial x} \gamma(1-\gamma) + \frac{m_1 m_2 \gamma(1-\gamma)(n_1 + n_2)}{[m_1 \gamma + m_2(1-\gamma)] p} \left( \frac{f_1}{m_1} - \frac{f_2}{m_2} \right) \right\}, \quad (4)$$

$$\alpha_p = \frac{m_1 - m_2}{m_1 \gamma + m_2(1-\gamma)}. \quad (5)$$

Equation (4) coincides with the analogous expression of [14], obtained using kinetic theory. However, it can easily be seen that since the quantities  $f_1$  and  $f_2$  are related to the pressure gradient  $\partial p/\partial x$  according to Eq. (3), this cumbersome expression can be reduced to the most simple form

$$v_1 - v_2 = -\frac{D_{12}}{\gamma(1-\gamma)} \left\{ \frac{\partial \gamma}{\partial x} - \alpha_f \frac{1}{p} \frac{\partial p}{\partial x} \gamma(1-\gamma) \right\}, \quad (6)$$

$$\alpha_f = \frac{f_1 - f_2}{f_1 \gamma + f_2(1-\gamma)}. \quad (7)$$

We note that from the viewpoint of calculating separation effects it is just the quantity  $\alpha_f$  which has physical meaning, since it defines the intensity of the mutual diffusion process induced by the absence of equilibrium between the "external" forces and the corresponding gradients in partial pressure of the components at times following the establishment of the full pressure gradient  $\partial p/\partial x$ . Equations (6) and (7) allow determination of the diffusion flow of the component  $j_1 = n_1 v_1 = \frac{n_1 n_2}{n_1 + n_2} (v_1 - v_2)$  in the presence within the system of a pressure gradient produced by different force fields. Moreover, we can obtain from Eqs. (6) and (7) with the assumption of insignificant spatial change in concentration  $\left( \frac{\gamma_A - \gamma_B}{\gamma(1-\gamma)} \ll 1 \right)$  an expression for the equilibrium separation coefficient

$$\beta = \left( \frac{\gamma}{1-\gamma} \right)_a / \left( \frac{\gamma}{1-\gamma} \right)_b \cong \exp \left( \bar{\alpha}_f \ln \frac{p_a}{p_b} \right). \quad (8)$$

We will define the quantities  $\bar{\alpha}_f$  and  $\beta$  for action on the separating mixture of concrete types of force field. In the case of forces proportional to the first power of the particle mass (gravitational or centrifugal), we have

$$f_1 = m_1 g^*, \quad f_2 = m_2 g^*, \quad (9)$$

$$\bar{\alpha}_f \cong \frac{m_1 - m_2}{m_1 \bar{\gamma} + m_2 (1 - \bar{\gamma})}, \quad \ln \beta \cong \frac{m_1 - m_2}{m_1 \bar{\gamma} + m_2 (1 - \bar{\gamma})} \ln \frac{p_a}{p_b}. \quad (10)$$

We note that the quantity  $\bar{\alpha}_f$ , described by Eq. (10), coincides with the barodiffusion constant calculated in [14, 15].

If the external forces are electromagnetic ones, acting, for example, on components of the mixture consisting of ions of two different sorts with an identical ionization level [16]:  $f_1 = f_2 = Ze(E_x + v_y B_z - v_z B_y)$ , then we obtain

$$\bar{\alpha}_f \equiv 0, \quad \ln \beta \equiv 0, \quad (11)$$

which implies the absence of mutual diffusion and separation effect in the given situation. However, when the charges of the ions differ in value ( $Z_1 \neq Z_2$ ), we find

$$\bar{\alpha}_f \cong \frac{Z_1 - Z_2}{Z_1 \bar{\gamma} + Z_2 (1 - \bar{\gamma})}, \quad \ln \beta \cong \frac{Z_1 - Z_2}{Z_1 \bar{\gamma} + Z_2 (1 - \bar{\gamma})} \ln \frac{p_a}{p_b}. \quad (12)$$

In this case the particles with higher charge are concentrated in that region of the separation zone toward which the action of the electromagnetic force is directed [2].

We will consider mutual diffusion in the presence of viscous forces. If the steady-state flow of the binary gas mixture occurs in a along planar channel under the action of a pressure head, we have [3]

$$f_1 \cong -\frac{2\eta_1}{n_1} \frac{\partial \epsilon_{xy}}{\partial y}, \quad f_2 \cong -\frac{2\eta_2}{n_2} \frac{\partial \epsilon_{xy}}{\partial y}. \quad (13)$$

Despite that fact that the given formulation of the problem assumes the absence of solid surfaces bounding the diffusion zone, Eq. (6) is also valid for this case, inasmuch as the pressure gradient is equalized by viscous friction on the channel wall. If for  $\eta_1$  and  $\eta_2$  we use the approximate expression (59) from [17], on the basis of Eqs. (7) and (13) we find

$$\bar{\alpha}_f \cong \frac{3}{4} \frac{\Delta m}{m}. \quad (14)$$

The value of  $\bar{\alpha}_f$  differs only insignificantly from the barodiffusion constant in a viscous flow, calculated in [3].

We will define the quantity  $\ln \beta$  for the case where the pressure gradient in the separating mixture is caused by diffusion friction forces produced by an auxiliary gas, diffusing through the isotopic mixture with a mean velocity  $U_N$ . Such a formulation of the problem assumes that the auxiliary gas can penetrate through the mixture being separated, for which the bounding surfaces are impermeable. In its most general features such an idealized situation is realized in a mass diffusion element, if for the third gas a vapor phase which condenses on the boundary surface is used [1]. In plasma devices the "auxiliary" gas is an ion component, which neutralizes near the cathode [10]. It should be noted that we deal here with a case of mutual diffusion for which the values  $f_1$  and  $f_2$  themselves, being diffusion friction forces, depend on the mean velocities  $v_1$  and  $v_2$ . If we neglect the effect of the finiteness of the velocities  $v_1$  and  $v_2$  on the forces  $f_1$  and  $f_2$  ( $v_1$  and  $v_2 \ll U_N$ ), we obtain with consideration of the results of [4]

$$f_1 \cong \frac{4}{3} N \bar{v}_{N1} \bar{m}_{N1} Q U_N, \quad (15)$$

$$f_2 \cong \frac{4}{3} N \bar{v}_{N2} \bar{m}_{N2} Q U_N. \quad (16)$$

In the example of mass-diffusion, the justifiability of introducing the term "external force" is clearly visible, since for the isotopic mixture being separated the forces  $f_1$  and  $f_2$  are purely external. Using Eqs. (7), (8) and (15), (16), we find

$$\bar{\alpha}_f \cong \frac{\Delta m}{2m} \frac{M_N}{M_N + m}, \quad \ln \beta \cong \frac{\Delta m}{2m} \frac{M_N}{M_N + m} \ln \frac{p_a}{p_b}.$$

Equation (8) permits estimation of the intensity of mutual diffusion for flow of a mixture in a capillary or planar slot, when the particle free path length significantly exceeds the transverse channel dimensions (Knudsen regime [7]). Assuming, for example, that the process of molecular reflection from the capillary walls is diffuse and the entire molecular momentum is lost on the surface  $\left(f_1 \simeq \sqrt{\frac{2kTm_1}{\pi R^2}} U_p, f_2 \simeq \sqrt{\frac{2kTm_2}{\pi R^2}} U_p\right)$ , we obtain  $\bar{\alpha}_f \simeq \frac{\Delta m}{2m}$ .

#### NOTATION

$p_1, p_2$  and  $n_1, n_2$ , partial pressures and densities of the components;  $p = p_1 + p_2$ ;  $m_1, m_2$ , particle masses;  $v_1, v_2$ , mean velocities;  $x$ , coordinate in the direction of which external forces act;  $a_{1,2}$ , diffusion friction coefficient [4];  $\gamma = n_1/(n_1 + n_2)$ ;  $D_{1,2}$ , mutual diffusion coefficient of mixture to be separated;  $p_a$  and  $p_b$ , mixture pressures in boundary zones of separation region;  $g^*$ , acceleration corresponding to "inertial" force;  $Z$ , ionic charge;  $E_x, B_y, B_z$ , components of electric field strength and magnetic induction;  $v_y, v_z$ , mean ion velocities in  $y$  and  $z$  directions;  $\epsilon_{xy}$ , deformation rate tensor component;  $\eta_1, \eta_2$ , "partial" viscosities of mixture components;  $\Delta m = m_1 - m_2$ ;  $\bar{m} = m_1\gamma + m_2(1 - \gamma)$ ;  $N$ , particle density of auxiliary gas;  $Q$ , effective diffusion section of elastic collisions between molecules of mixture to be separate and auxiliary gas:  $\bar{m}_{N1} = (M_N m_1)/(M_N + m_1)$ ;  $v_{N1} = \sqrt{8kT/\pi \bar{m}_{N1}}$ ;  $T$ , temperature;  $U_e$ , mean velocity of Knudsen flow components in equilibrium;  $R$ , capillary radius;  $\alpha_f = f_1 - f_2/[f_1\bar{\gamma} + f_2(1 - \bar{\gamma})]$ ;  $\bar{\gamma} = (\gamma_A + \gamma_B)/2$ .

#### LITERATURE CITED

1. A. M. Rozen, Isotope Separation Theory in Columns [in Russian], Atomizdat, Moscow (1960).
2. S. I. Braginskii, "Protective rotating layer on the boundary of a high temperature plasma," *Fiz. Plazmy*, 1, No. 3, 370-377 (1975).
3. V. M. Zhdanov, Yu. M. Kagan, and A. A. Sazykin, "Effect of viscous momentum transfer on diffusion in a gas mixture," *Zh. Eksp. Teor. Fiz.*, 42, No. 3, 857-867 (1962).
4. V. M. Zhdanov, A. I. Karchevskii, E. P. Potanin, and A. L. Ustinov, "The effect of 'ion wind' on separation in a plasma centrifuge with crossed fields," *Zh. Tekh. Fiz.*, 49, No. 9, 1879-1883 (1979).
5. A. I. Karchevskii and E. P. Potanin, "Separation of gas mixtures in systems with a traveling magnetic field," *Zh. Tekh. Fiz.*, 48, No. 10, 2097-2102 (1978).
6. E. P. Potanin, "Component transport equation in a column in the presence of longitudinal separation," *Inzh.-Fiz. Zh.*, 39, No. 1, 81-85 (1980).
7. V. D. Seleznev, P. E. Suetin, and N. A. Smirnov, "Separation of a binary gas mixture over the entire Knudsen number range," *Zh. Tekh. Fiz.*, 45, No. 7, 1499-1502 (1975).
8. A. I. Karchevskii, E. P. Potanin, A. A. Sazykin, and A. L. Ustinov, "Separation of isotopic mixture in a rotating plasma," *Fiz. Plazmy*, 8, No. 2, 306-311 (1982).
9. A. V. Belorusev, A. I. Karchevskii, Yu. A. Muromkin, and E. P. Potanin, "Separation of gas mixtures in xenon isotopes in an impulsive plasma centrifuge," *Pis'ma Zh. Tekh. Fiz.*, 2, No. 14, 664-668 (1976).
10. V. M. Zhdanov, A. I. Karchevskii, and E. P. Potanin, "On the effect of 'ion wind' on separating properties of a plasma centrifuge with crossed fields," *Zh. Tekh. Fiz.*, 4, No. 9, 508-511 (1978).
11. A. I. Karchevskii and E. P. Potanin, "On calculation of thermodiffusion processes in a system with traveling magnetic field," *Inzh.-Fiz. Zh.*, 37, No. 1, 72-75 (1979).
12. Y. Matsumura and T. Abe, "Neon isotope isolation by cataphoresis in a dc gas discharge," *Jpn. J. Appl. Phys.*, 19, 457-459 (1980).
13. A. I. Karchevskii and E. P. Potanin, "Isotope separation in dc discharges," *Fiz. Plazmy*, 8, No. 1, 178-181 (1982).
14. S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases*, Cambridge Univ. Press (1970).
15. A. G. Shashkov and T. N. Abramenko, *Cross Effects in Gas Mixtures* [in Russian], Nauka i Tekhnika, Minsk (1976).
16. O'Neil, "Centrifugal separation of a multispecies pure ion plasma," *Phys. Fluids*, 24, No. 8, 1447-1462 (1981).
17. I. F. Golubev and N. E. Gnezdilov, *Viscosity of Gas Mixtures* [in Russian], State Standards Committee, Soviet of Ministers of the USSR, Moscow (1971).